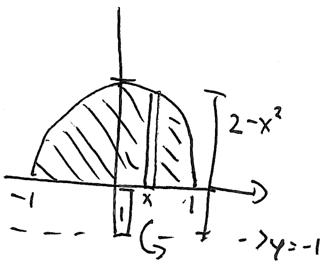
MA114 Summer 2018
Worksheet 18 – Volumes II – 7/16/18

1. Use the method of disks/washers to find the volume of the solid of revolution generated by rotating the region bounded by $y = 1 - x^2$ and $y = 0$ about the line $y = -1$.



$$V = \pi \int_{-1}^{1} (2-x^2)^2 - 1^2 \, dx$$

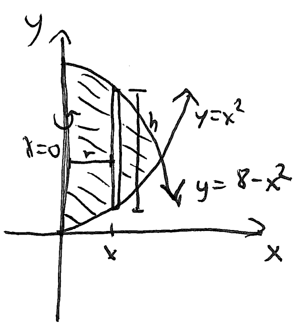
$$= \pi \int_{-1}^{1} 4 - 4x^2 + x^4 - 1 \, dx$$

$$= \pi \left(3x - \frac{4}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1}$$

$$= 2\pi\left(3 - \frac{4}{3} + \frac{1}{5}\right)$$

$$= 2\pi\left(\frac{45-20+3}{15}\right) = 2\pi \cdot \frac{28}{15} = \boxed{\frac{56}{15}\pi}$$

2. Use the method of cylindrical shells to find the volume of the solid of revolution generated by rotating the region bounded by $y = x^2, y = 8 - x^2$, and $x = 0$ for $x \geq 0$ around the y-axis.



$r = x$

$h = (8-x^2) - x^2$

$= 8 - 2x^2$

intersection: $x^2 = 8 - x^2$

$2x^2 = 8$

$x^2 = 4$

$x = 2$

$$V = 2\pi \int_0^2 x(8-2x^2)\,dx$$

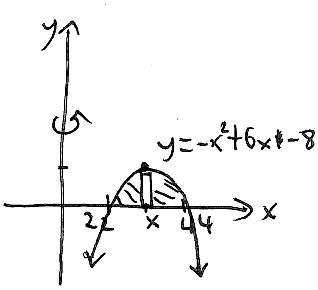
$$= 2\pi \int_0^2 8x - 2x^3 \, dx$$

$$= 2\pi\left(4x^2 - \frac{x^4}{2}\right)\Big|_0^2$$

$$= 2\pi(16 - 8) = \boxed{16\pi}$$

3. Find an integral expression for the volume of the solid generated by rotating the given region R about the specified axis using any method.

a) R is the region bounded by $y = -x^2 + 6x - 8, y = 0$ about the y-axis.



$y = -(x^2 - 6x + 8) = -(x-4)(x-2)$

Use shells: $r = x$, $h = y = -x^2 + 6x - 8$

$$V = 2\pi \int_2^4 x(-x^2 + 6x - 8)\,dx = 2\pi \int_2^4 -x^3 + 6x^2 - 8x \, dx$$

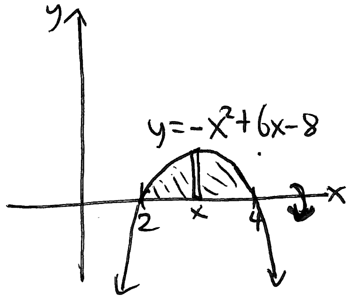
$$= 2\pi\left(-\frac{x^4}{4} + 2x^3 - 4x^2\right)\Big|_2^4$$

$$= 2\pi\left(-4^3 + 2\cdot 4^3 - 4^3 + 4 - 2^4 + 4\cdot 4\right)$$

$$= \boxed{8\pi}$$

b) R is the region bounded by $y=-x^2+6x-8, y=0$ about the x-axis.

Same region as a)



Use disks/washers:

$R_{in}=0$

$R_{out}=-x^2+6x-8$

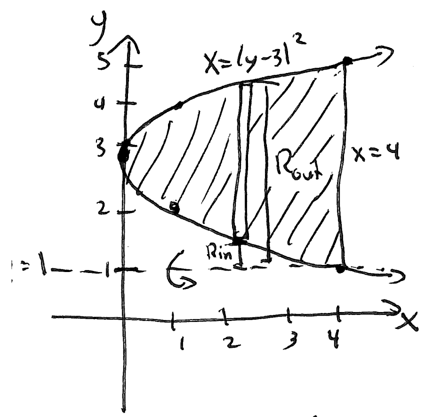
$V=\int_2^4 \pi((-x^2+6x-8)^2-0^2)\,dx$

$=\pi\int_2^4 x^4-12x^3+\cancel{28}\cdot 52x^2-96x+64\,dx$

$=\pi\left(\frac{x^5}{5}-3x^4+\frac{52}{3}x^3-48x^2+64x\right)\Big|_2^4$

$=\boxed{\frac{16\pi}{15}}$

c) R is the region bounded by $x=(y-3)^2, x=4$, about $y=1$.



~~Use shells to avoid~~ Either method works.

For disks/washers: Solve $x=(y-3)^2$ for y: $y=3\pm\sqrt{x}$

So $R_{in}=(3-\sqrt{x})-1=2-\sqrt{x}$, $R_{out}=(3+\sqrt{x})-1=2+\sqrt{x}$.

Then $V=\pi\int_0^4 R_{out}^2-R_{in}^2\,dx=\pi\int_0^4 (2+\sqrt{x})^2-(2-\sqrt{x})^2\,dx$

$=\pi\int_0^4 4+2\sqrt{x}+x-(4-2\sqrt{x}+x)\,dx$

$=\pi\int_0^4 4\sqrt{x}\,dx$

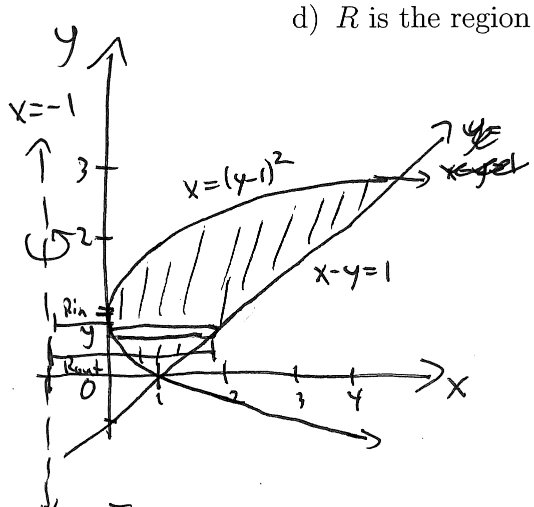
$=4\pi\cdot\frac{2}{3}x^{3/2}\Big|_0^4=\frac{8\pi}{3}\cdot 4^{3/2}=\boxed{\frac{64\pi}{3}}$

Intersection: $4=(y-3)^2$

$\pm2=y-3$

$y=5,1$

d) R is the region bounded by $x=(y-1)^2, x-y=1$, about $x=-1$.



Disks/Washers easiest to avoid splitting integral at $x=1$:

$R_{in}=(y-1)^2-(-1)=(y-1)^2+1=y^2-2y+2$

$R_{out}=(1+y)-(-1)=2+y$

$V=\pi\int_0^3 (2+y)^2-(y^2-2y+2)^2\,dy$

$=\pi\int_0^3 (4+4y+y^2)-(y^4-4y^3+8y^2-8y+4)\,dy$

$=\pi\int_0^3 -y^4+4y^3-7y^2+12y\,dy$

$=\pi\left(-\frac{y^5}{5}+y^4-\frac{7}{3}y^3+6y^2\right)\Big|_0^3$

$=\pi\left(-\frac{3^5}{5}+3^4-\frac{7}{3}\cdot 3^3+6\cdot 3^2\right)=\boxed{\frac{117\pi}{5}}$

Intersections:

$(y-1)^2=1+y \to y^2-3y=0 \to y=0, y=3$

$y^2-2y+1=1+y \quad y(y-3)=0$